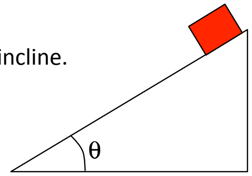
Problem 5.43

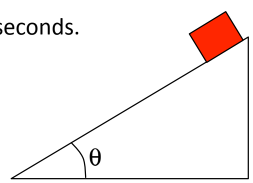
The block starts from rest at the top of the thirty-degree incline.



NOTE:

In most cases, N.S.L. is used when you are given very primary information about a system (the mass of the pieces in the system, maybe a coefficient of friction and the geometry—in this case its an incline with a known angle). What you are usually asked to determine is what is referred to as *the acceleration of the system*. For whatever reason, your book has generated a slew of problems in which you have to use kinematics to determine the acceleration, then use that acceleration and N.S.L. to determine other things (in this case, they are angling for the coefficient of friction). There is nothing wrong with this from an educational standpoint. It is, though, somewhat odd from a "conventional physics" perspective.

Again, the car accelerates through a quarter mile in 4.43 seconds.



a.) What is the coefficient of friction between the block and the incline?

We need to begin by determining the acceleration using kinematics. Taking a line along the incline and identifying it as the x-axis, we can write:

$$x_2 = \cancelto{0}{x_1} + \cancelto{0}{v_1}(\Delta t) + \frac{1}{2}a(\Delta t)^2$$

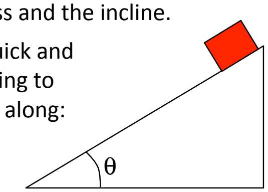
$$\Rightarrow \quad a = \frac{2x_2}{(\Delta t)^2}$$

$$= \frac{2(2.0 \text{ m})}{(1.5 \text{ s})^2}$$

$$= 1.78 \text{ m/s}^2$$

b.) Determine the coefficient of friction between the mass and the incline.

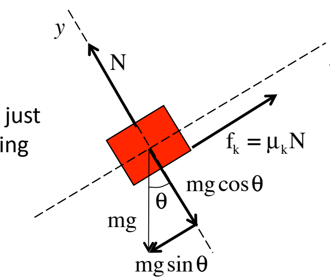
This is a N.S.L. problems and, as usual, we can do it Quick and Dirty or using the Formal approach. In fact, we are going to combine to two (a little of this, a little of that). Follow along:



A f.b.d., complete with axes and off-axis forces split into components, is shown below.

Note 1: I'll take forces that accelerate the mass UP the incline to be positive, and those DOWN the incline to be negative. With that model, the acceleration (DOWN the incline) will be negative.

Note 2: We will need the normal force. We could just eyeball from the f.b.d., but if that isn't obvious, using N.S.L. in the y-direction yields:

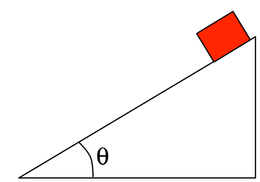


$$\underline{\sum F_y:}$$

$$-mg\cos\theta + N = m\cancelto{0}{a_y}$$

$$\Rightarrow \quad N = mg\cos\theta$$

With the information summarized in the "Notes" on the previous page, the forces that are motivating the system to accelerate are:



$$\underline{\sum F_{acc}:}$$

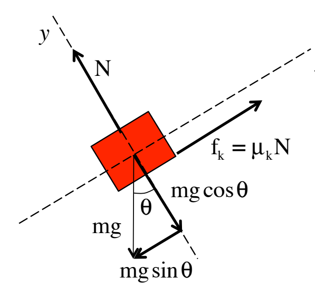
$$-mg\sin\theta + \mu_k N = -ma$$

$$\Rightarrow \quad -\cancel{m}g\sin\theta + \mu_k(\cancel{m}g\cos\theta) = -\cancel{m}a$$

$$\Rightarrow \quad \mu_k = \frac{g\sin\theta - a}{g\cos\theta}$$

$$= \frac{(9.8 \text{ m/s}^2)\sin 30^\circ - (1.78 \text{ m/s}^2)}{(9.80 \text{ m/s}^2)\cos 30^\circ}$$

$$= .368$$



c.) The frictional force acting on the mass:

$$\begin{aligned} f_k &= \mu_k N \\ &= \mu_k (mg\cos\theta) \\ &= (.368)(3.00\ \text{kg})(9.80\ \text{m/s}^2)\cos 30^\circ \\ &= 9.37\ \text{N} \end{aligned}$$

d.) The speed of the block at the bottom?

This is most easily done with energy considerations, but for now we will have to use kinematics:

$$v_{x,2} = \cancelto{0}{v_{x,1}} + a_x \Delta t$$

$$\Rightarrow \quad v_{x,2} = (1.78\ \text{m/s}^2)(1.50\ \text{s}) = 2.67\ \text{m/s}$$

OR

$$(v_{x,2})^2 = (\cancelto{0}{v_{x,1}})^2 + 2a\Delta x$$

$$\Rightarrow \quad v_{x,2} = \sqrt{2(1.78\ \text{m/s}^2)(2.00\ \text{m})} = 2.67\ \text{m/s}$$